MATHEMATICS METHODS

MAWA Semester 1 (Unit 3) Examination 2019
Calculator-free

Marking Key

The release date for this exam and marking scheme is 14th June.

Section One: Calculator-free (50 Marks)

Question 1(a) (2 marks)

Solution	
$Let\ f(x) = xe^{3x}$ $f'(x) = e^{3x} + x3e^{3x} = e^{3x} + 3xe^{3x}$	
Mathematical behaviours	**Mark**
• applies product rule	1
• differentiates exponential correctly	1

Question 1(b) (3 marks)

Solution	
$\frac{d}{dx}\left(\frac{\cos x}{x^3}\right) = \frac{x^3(-\sin x) - \cos x(3x^2)}{(x^3)^2} = \frac{-x^2(x\sin x + 3\cos x)}{x^6} = -\frac{(x\sin x + 3\cos x)}{x^4}$	
Mathematical behaviours	**Marks**
• applies quotient rule	1
• differentiates $\cos x$ correctly	1
• simplifies result	1

Question 1(c) (3 marks)

Solution	
$g(u) = \sqrt{u} \Rightarrow \frac{dg}{du} = \frac{1}{2}u^{\frac{-1}{2}}$ $\qquad u = 2 - 3x^2 \Rightarrow \frac{du}{dx} = -6x$ $\Rightarrow \frac{dg}{dx} = \frac{1}{2}u^{\frac{-1}{2}} \times -6x = \frac{-3x}{\sqrt{2-3x^2}}$	
Mathematical behaviours	**Marks**
• states $\frac{dg}{du}$	1
• states $\frac{du}{dx}$	1
• states $\frac{dg}{dx}$ in terms of x.	1

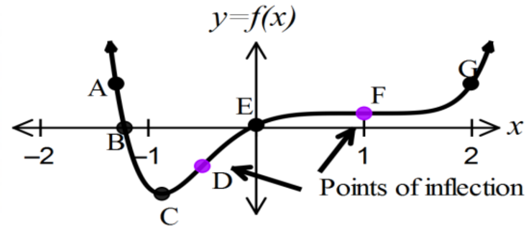
Question 1(d) (3 marks)

Solution	
$x(t) = 3\sin 2t \Rightarrow v(t) = 3 \times 2\cos 2t$ $v(t) = 0 \Rightarrow \cos 2t = 0$ $ie\ 2t = \frac{\pi}{2} \Rightarrow t = \frac{\pi}{4} s$	
Mathematical behaviours	**Marks**
• differentiates to obtain $v(t)$	1
• equates $v(t) = 0$	1
• determines t value	1

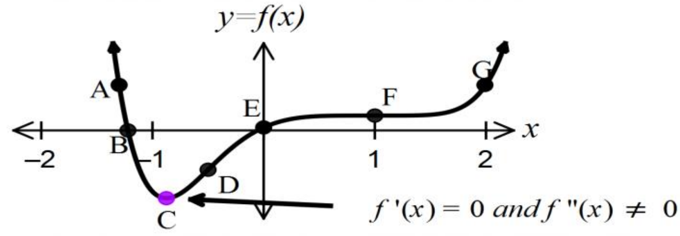
Question 2 (a) **(4 marks)**

Solution

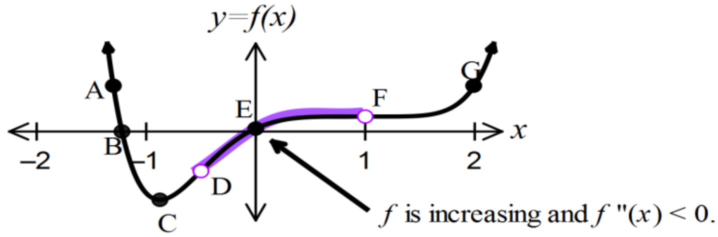
(i)



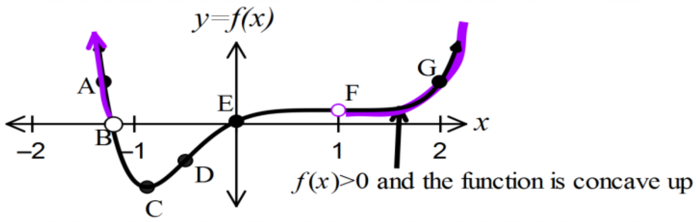
(ii)



(iii)



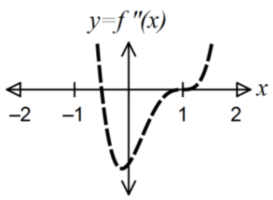
(iv)



Mathematical behaviours	Mark
(i)	
• states D and F	1
(ii)	
• states C	1
(iii)	
• states E	1
(iv)	
• states A and G	1

Question 2(b) **(1 mark)**

Solution

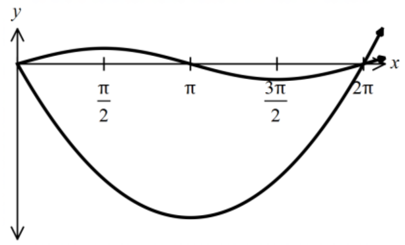


Mathematical behaviours	Marks
• circles the 2nd graph	1

Question 3 **(4 marks)**

Solution

Area = $\int_0^{2\pi}\left[\sin x - x\left(x-2\pi\right)\right]dx$



$= \int_0^{2\pi}(\sin x - x^2 + 2\pi x)dx$

$= \left[-\cos x - \frac{x^3}{3} + \pi x^2\right]_0^{2\pi}$

$= \left[-\cos 2\pi - \frac{(2\pi)^3}{3} + \pi(2\pi)^2\right] - \left[-\cos 0\right]$

$= -1 - \frac{8\pi^3}{3} + 4\pi^3 + 1$

$= \frac{4\pi^3}{3}$

Mathematical behaviours	Marks
• states a correct expression using integrals to determine the area	1
• anti-differentiates each part correctly	1
• substitutes in limits of integration	1
• evaluates result	1

Question 4(a) **(2 marks)**

Solution
$\int\left(2e^{2x}-\frac{3}{\sqrt{x}}\right)dx=\int\left(2e^{2x}-3x^{\frac{-1}{2}}\right)dx$ $=2\frac{e^{2x}}{2}-3\frac{x^{\frac{1}{2}}}{\frac{1}{2}}$ $=e^{2x}-6\sqrt{x}+c$

Mathematical behaviours	Marks
• anti-differentiates the exponential function correctly	1
• anti-differentiates the square root function correctly	1

Question 4(b) **(2 marks)**

Solution
$\int_0^1 (3-2x)^2\,dx$ $=\left[\frac{(3-2x)^3}{3\times(-2)}\right]_0^1$ $=\frac{-1}{6}\left(1^3-3^3\right)$ $=\frac{26}{6}$ $=\frac{13}{3}$

Mathematical behaviours	Marks
• aniti-differentiates correctly	1
• substitutes limits of integration and evaluates	1

Question 4(c) **(2 marks)**

Solution
$F(x)=\int_x^1 \frac{dt}{1+\sqrt{1-t}}$ $=-\int_1^x \frac{dt}{1+\sqrt{1-t}}$ $\therefore F'(x)=-\frac{1}{1+\sqrt{1-x}}$

Mathematical behaviours	Marks
• uses the relationship $\int_x^1 \frac{dt}{1+\sqrt{1-t}} = -\int_1^x \frac{dt}{1+\sqrt{1-t}}$	1
• applies Fundamental Theorem of Calculus	1

Question 4(d) **(3 marks)**

Solution

$$\int_{-m}^{m} \left(m^3 - x^3\right) dx = 1250$$

$$\left[m^3x - \frac{x^4}{4}\right]_{-m}^{m} = 1250$$

$$\left[m^4 - \frac{m^4}{4}\right] - \left[-m^4 - \frac{m^4}{4}\right] = 1250$$

$$\frac{3m^4}{4} + \frac{5m^4}{4} = 1250$$

$$\frac{8m^4}{4} = 1250$$

$$2m^4 = 1250$$

$$m^4 = 625$$

$$m = \pm 5$$

Mathematical behaviours	Marks
• anti-differentiates integral correctly	1
• substitutes in limits of integration correctly and simplifies to obtain correct expression on the LHS	1
• determines correct answers for m.	1

Question 5(a) **(3 marks)**

Solution

Bernoulli distribution with $\mu = \frac{1}{36}$, $\sigma^2 = \frac{1}{36} \times \frac{35}{36} = \frac{35}{36^2}$

Mathematical behaviours	Marks
• states Bernoulli	1
• states mean	1
• states variance	1

Question 5(b) **(3 marks)**

Solution

This represents a Binomial with $n = 15$ and $p = \frac{1}{36}$.

Mathematical behaviours	Marks
• states Binomial	1
• states n	1
• states p	1

Question 5(c) **(1 mark)**

Solution	
$W \sim Bin\left(15, \frac{1}{36}\right)$ $P(W=1) = {}^{15}C_1 \left(\frac{1}{36}\right)^1 \left(\frac{35}{36}\right)^{14} = 15\left(\frac{1}{36}\right)^1 \left(\frac{35}{36}\right)^{14}$	
Mathematical behaviours	**Marks**
• states correct expression	1

Question 5(d) **(3 marks)**

Solution	
$Let\ Z \sim Bin\left(30, \frac{1}{36}\right)$ $P(Z=2 \mid Z \geq 1) = \frac{P(Z=2)}{P(Z \geq 1)} = \frac{P(Z=2)}{1-P(Z=0)} = \frac{{}^{30}C_2 \left(\frac{1}{36}\right)^2 \left(\frac{35}{36}\right)^{28}}{1 - {}^{30}C_0 \left(\frac{1}{36}\right)^0 \left(\frac{35}{36}\right)^{30}}$	
Mathematical behaviours	**Marks**
• recognises the situation involves a binomial $(30, \frac{1}{36})$ and conditional probability • states correct expression for numerator • states correct expression for denominator	1 1 1

Question 6(a) **(1 mark)**

Solution	
(i) Under-estimated Area = $\left(\frac{\pi}{6} \times \frac{1}{2}\right) + \left(\frac{\pi}{6} \times \frac{\sqrt{3}}{2}\right)$ $= \frac{\pi}{6}\left(\frac{1+\sqrt{3}}{2}\right)$ (ii) Over-estimated Area = $\left(\frac{\pi}{6} \times \frac{1}{2}\right) + \left(\frac{\pi}{6} \times \frac{\sqrt{3}}{2}\right) + \left(\frac{\pi}{6} \times 1\right)$ $= \frac{\pi}{6}\left(\frac{1}{2} + \frac{\sqrt{3}}{2} + 1\right)$ $= \frac{\pi}{6}\left(\frac{3+\sqrt{3}}{2}\right)$	
Mathematical behaviours	**Marks**
(i) • states the sum of the area of the two rectangles and simplifies correctly (ii) • states the sum of the area of the three rectangles and simplifies correctly	 1 1

Question 6(b) **(2 marks)**

Solution
Using trapeziums is equivalent to averaging the results from part (a) i.e. Estimated area under $f(x) = \sin x$ from $x = 0$ to $x = \frac{\pi}{2}$ is $\left[\frac{\pi}{6}\left(\frac{3+\sqrt{3}}{2}\right) + \frac{\pi}{6}\left(\frac{1+\sqrt{3}}{2}\right)\right] \div 2$ $= \left[\frac{\pi}{6}\left(\frac{4}{2} + \frac{2\sqrt{3}}{2}\right)\right] \div 2$ $= \left[\frac{\pi}{6}\left(2+\sqrt{3}\right)\right] \div 2$ $= \frac{\pi}{6}\left(\frac{2+\sqrt{3}}{2}\right)$

Mathematical behaviours	Marks
• determines the average of the two areas obtained in part (a) • simplifies to deduce the required result	1 1

Question 7(a) **(1 mark)**

Solution
$y = \sin^2 x$ $\frac{dy}{dx} = 2\sin x \cos x$

Mathematical behaviours	Mark
• States correct answer	1

Question 7(b) **(3 marks)**

Solution
$y = \sin^2 x$ $\frac{dy}{dx} = 2\sin x \cos x$ $\int \frac{dy}{dx}\,dx = \int 2\sin x \cos x\,dx$ $y = \int 2\sin x \cos x\,dx + c$ $\sin^2 x = \int 2\sin x \cos x\,dx + c$ $\int \sin x \cos x\,dx = \frac{1}{2}\sin^2 x + c$

Mathematical behaviours	Marks
• integrates both sides of equation • applies fundamental theorem • rearranges to get required result	1 1 1

Question 7(c) **(3 marks)**

Solution	
$\int_0^{\frac{\pi}{6}} (\sin x \cos x + 2)\, dx = \left[\frac{1}{2}\sin^2 x + 2x\right]_0^{\frac{\pi}{6}}$ $= \frac{1}{2}\left[\left(\frac{1}{2}\right)^2 - 0^2\right] + 2\left[\frac{\pi}{6} - 0\right]$ $= \frac{1}{8} + \frac{\pi}{3}$	
Mathematical behaviours	**Marks**
• recognises $\sin^2 x$ term is to be involved • states correct integral and bounds of integration • substitutes bounds of integration and simplifies	1 1 1